

Supergalactic Structure of Energy-Angle Correlations

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Evidence for the supergalactic structure of multiplets (energy-angle correlations) has previously been shown using ultra-high energy cosmic ray (UHECR) data from Telescope Array (TA) with energies above 10^{19} eV. The supergalactic deflection hypothesis (that UHECR sources and intervening magnetic fields are correlated) is measured by the all-sky behavior of the strength of intermediate-scale correlations. The multiplets are measured in spherical surface wedge bins of the field-of-view to account for uniform and random magnetic fields. The structure found is consistent with the previously published energy spectrum anisotropy results of TA and toy-model simulations of a supergalactic magnetic sheet. The 7 year data post-trial probability of this feature appearing by chance, on an isotropic sky, was found by Monte Carlo simulation to be ∼4σ. The analysis has now been applied to 10 years of data.

36th International Cosmic Ray Conference -ICRC2019- July 24th - August 1st, 2019 Madison, WI, U.S.A.

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1. Introduction

Large scale magnetic fields have been measured between clusters of galaxies which make up the supergalactic plane (the average matter distribution largely within the GZK cutoff of 100 Mpc) such as the Coma Cluster ([[1](#page-8-0)], [\[2\]](#page-8-0)). Recently a \sim 3 Mpc field between Abell 0399 and Abell 0401 has been observed that acts as a particle reaccelerator [\[3\]](#page-8-0). It has also been shown that ∼90% of the baryonic mass of the universe is between galaxies ([\[4\]](#page-8-0)) that these may support formation of even larger intra-galactic scale magnetic fields ([\[5\]](#page-8-0),[[6](#page-8-0)]).

Large scale magnetic fields suggest that energy dependent deflection of ultra-high energy cosmic rays (UHECR) may appear correlated with the SGP. Previous energy-position correlation (multiplet) searches for small scale magnetic deflections have not had significant results ([[7](#page-8-0)],[\[8](#page-8-0)],[\[9\]](#page-8-0)). This analysis uses intermediate-scale energy-position correlations to look for significant large scale magnetic structure. We report an update using ten years of Telescope Array data from the seven year result reported in [[10\]](#page-8-0) though the significance has not been updated.

2. Data Set

SD data recorded between May 11 of 2008 and 2019 is used for this analysis. The 2016 data has been removed due to SD array communication tower issues that caused a very high day to day variance, in the variance between trigger times each day in the year. This introduced non-physical anisotropies to the data that are non-trivial to compensate for. The energy of reconstructed events is determined by the SD array and renormalized by 1/1.27 to match the calorimetric fluorescence detector energy scale ([[11\]](#page-8-0)). The reconstruction of these events is the same as the "Hotspot" anal-ysis of [\[12](#page-8-0)]. Due to the use of lower energies down to $10^{19.0}$ eV tighter data cuts are required for good zenith angle and energy resolution.

After cuts, were 3027 events in the seven year data set and there are 4321 using ten years of data. Events in the data set match the following criteria:

- 1. E≥10^{19.0} eV (where detection efficiency is ~100%).
- 2. At least four SDs triggered.
- 3. Zenith angle of arrival direction $<$ 55 $^{\circ}$.
- 4. Reconstructed pointing direction error $\langle 5^\circ \rangle$.
- 5. Shower core >1.2 km from array boundary.
- 6. Shower lateral distribution fit χ^2/dof < 10.

The additional cuts on pointing direction error and boundary distance improve the agreement between the distribution of zenith angles compared to the geometrical zenith angle exposure $g(\theta)$ = $sin(\theta)cos(\theta)$. The azimuthal angle distribution is in agreement with a flat distribution. The energy spectrum is also in good agreement with the published spectrum ([[11\]](#page-8-0),[[13](#page-8-0)]).

Energy resolution and zenith angle resolution of events range from \sim 10 to 15% and \sim 1.0° to 1.5◦ , respectively, depending on core distance from the array boundary and improve with increasing energy. These resolutions are sufficient to search for UHECR energy anisotropies.

3. Energy-Angle Correlations

It's assumed that UHECR travel through uniform fields, Equation 3.1a, and random fields where the deflection variance is shown in Equation 3.1b (*Z* is mass number, *S* distance, *B* field strength, *E* particle energy, and *L^c* is mean magnetic field coherence length). The result is lower energy cosmic ray events are deflected to larger angles from their source than higher energy events [[14\]](#page-9-0). Figure 1 shows a diagram of this energy dependent drift-diffusion.

Figure 1: UHECR Drift-diffusion deflection. (a) Two events having traveled through uniform and random magnetic fields. The purple vector represents the low energy event spherical arc and the red vector is a higher energy event. Uniform and random magnetic field components describe the average field perpendicular to the field of view (FOV) sphere. Possible random field deflections are represented by dashed circles. (b) A spherical cap section, "wedge," is a simple shape that best encompasses the likeliest positions. Pointing direction is the spherical arc ϕ , width is $\Delta \phi$, and *D* is the maximum angular distance.

3.1 Correlation

Correlations between energy and great circle angular distance of events are found using a ranked correlation, Kendall's τ_b , that measures the strength of monotonic dependence [\[15](#page-9-0)]. Ranked correlation is the linear correlation between the sorted individual ordering of the two variables of interest (Variable *x*: 1st, 2nd, 3rd, etc. versus Variable *y*: 5th, 1st, 4th, etc.). If $\tau_b = -1$ an increase (*decrease*) of *x* always follows a decrease (*increase*) in *y*. For +1 an increase (*decrease*) of *x* always follows an increase (*decrease*) of *y*. This removes magnetic model and composition assumptions and also detector exposure effects on the correlation strength.

The significance of a correlation (probability of $\tau_b=0$) is a function of correlation strength and sample size. This is found by permutations of the sample or the large sample limit that follows the standard normal distribution. Further details can be found in [[10\]](#page-8-0) and [\[15](#page-9-0)].

3.2 Correlation Binning

Possible UHECR deflections from sources were found by a scanned maximization of the significance of energy-angle correlations inside spherical cap sections, or "wedges," using seven years of Telescope Array data [[10\]](#page-8-0). An example is shown in Figure [2\(a\).](#page-3-0) This was done at each point on an equal 2[°] spaced grid on the FOV. Wedge bins are defined by a maximum angle radius, δ_j , from

grid point, *i*, and the boundaries of two azimuths (Equation 3.2a). The angular distance between the wedge pointing direction, ϕ_i , and an events azimuth is given by Equation 3.2b.

$$
\phi_{ij} = \operatorname{atan} \frac{\cos B_i \sin (L_i - L_j)}{\cos B_j \sin B_i - \sin B_j \cos B_i \cos (L_i - L_j)}
$$
(3.2a)

$$
\Delta \phi_{ij} = \text{mod} \left(|\phi_{ij} - \phi_i| + 180, 360 \right) - 180 \tag{3.2b}
$$

This bin shape requires four parameters to be scanned at every grid point to maximize the significance. Though negative correlations are physically expected from magnetic deflections; the sign of the correlation, and its strength, are not scanned for nor restricted. Reasonable deflection scenarios are taken into account by a large parameter space and are the following:

- 1. Energy Threshold, *Eⁱ* : 10 to 100 EeV, 5 EeV steps.
- 2. Wedge Angular Distance, $D_i = \max(\delta_{ij})$: 15° to 90°, 5° steps.
- 3. Wedge Direction, ϕ_i : 0 \degree to 355 \degree , 5 \degree steps.
- 4. Wedge Width, $W_i = 2 \max(|\Delta \phi_{ij}|)$: 10[°] to 90[°], 10[°] steps (5[°] on each side of ϕ_i).

Events are in the wedge if $E_j \ge E_i \& \delta_{ij} \le D_i \& -W_i/2 \le \Delta \phi_{ij} \le W_i/2$ (*i* is the grid point). The parameters (E_i, D_i, ϕ_i) , and W_i) are chosen for the minimum p-value of the energy-angle correlation, $\tau_b(\delta_i, E_i)$ (maximum significance).

3.3 Most Significant Correlation

These parameters were scanned for using seven years of Telescope Array data as shown in [[10\]](#page-8-0). No parameters were changed for the ten year data update shown here. The most significant correlation in 10 years of data is at 30.3◦ SGB, -3.2◦ SGL, 10◦ from M82 pointing towards the TA Hotspot, and shown in Figure 2(a). With 75 events (E≥35 EeV) τ_b =−0.412 has pre-trial significance of 5.10 σ . This is an increase from 4.58 σ at this grid point with 7 years of data using the same wedge and energy threshold parameters. Figure 2(b) shows a scatter plot of energy versus angular distance and a linear fit (Equation [3.1a](#page-2-0) with $Z=1$) results in an estimate of $S \times B = 41.1$ kpc $\times\mu$ G. If the source is assumed to be at the distance to M82 (3.7 Mpc) the average uniform magnetic field required to cause this deflection would be $B=12$ nG.

Figure 2: (a) Supergalactic Hammer-Aitoff projection of the 10 year maximum significance "wedge" at 30.3◦ SGB, -3.2◦ SGL. The correlation τ*b*=−0.412 with 75 data events has a pre-trial one-sided significance of 5.10 σ . This increased from 4.58 σ at this grid point with 7 years of data. The energy threshold is $E_i \ge 35$ EeV, wedge width $W_i = 90^\circ$, angular distance $D_i = 70^\circ$, and direction $\phi_i = 120^\circ$. (b) Scatter plot of $1/E_j$ versus angular distance δ_j in the wedge.

4. Simulations

The analysis was applied to isotropic simulations for the seven year anisotropy significance calculation as described further in Section [5](#page-5-0). A second simulation is a simple simulation of a supergalactic magnetic sheet resulting in an energy dependent diffusion of events away from the supergalactic plane. This is used to motivate the test statistic that was used to test the hypothesis of supergalactic sources and magnetic fields.

4.1 Isotropic Simulation

Actual data coordinates were used for the isotropic simulations. New energies were assigned to data positions by interpolating a large set of MC energies reconstructed through a surface detector simulation thrown with the HiRes/TA spectrum ([\[16](#page-9-0)], [[17\]](#page-9-0)). The number of events in each isotropic MC event set were the same as data in each 5 EeV bin. This simulated the expected data given the detector configuration and on-time with no energy anisotropies.

4.2 Supergalactic Magnetic Sheet

A simple toy-model of a supergalactic magnetic sheet uses event deflections in SGB, proportional to 1/*energy* according to Equation [3.1](#page-2-0)a, for a fraction of events in the isotropic simulations. A simulation, with an $F = 65.7\%$ isotropic fraction (1988 out of 3027 events) and $S \times B = 18.47$ $kpc \times \mu$ G, is shown in Figure 3(a). This is only an anisotropy of energies, event positions are isotropic.

Figure 3: Toy-model supergalactic magnetic sheet simulation of anisotropic energies. (a) Blue circles are isotropic events $(F = 65.7\%)$. Red squares are anisotropic events magnetically diffused away from the supergalactic plane with $S \times B = 18.47$ kpc $\times \mu$ G. All event positions are isotropic and the energy spectrum is created according to the published TA results. (b) Projection of the energy-angle correlation strength τ_b inside wedges for all grid points. Solid curves indicate the galactic plane (GP) in blue and supergalactic plane (SGP) in red.

Unlike the isotropic simulations data coordinates are not used. On-time is simulated by sampling the trigger times of 264,499 data events with $E > 10^{17.7}$ eV. The azimuthal angle distribution is 0° to 360° uniform and the zenith angle distribution is $g(\theta) = \sin(\theta)\cos(\theta)$ due to the flat SD array. Positions of energies are moved by δ_k , for each event, *j*, in the anisotropic fraction assuming protons (*Z*=1), and an *S*×*B*. The energies, E_k , and their deflection from *SGB* = 0 (δ_k) are assigned to the closest isotropic SGB_j value (min $[\delta_k$ - $SGB_j]$) in random order. Further detail is in [\[10\]](#page-8-0)

It can be seen via this simple model in the projection of the τ_b , in Figure [3\(b\)](#page-4-0), that if there are magnetically induced energy-angle correlations clustered in the supergalactic plane, negative correlation wedges will be close to the supergalactic plane. Furthermore, since negative correlations viewed from the opposite direction appear as a positive correlations $((x_j, y_j) \rightarrow (x_j, -y_j))$, positive correlations are expected at large distances from the supergalactic plane.

5. Supergalactic Structure

No single correlation tests the hypothesis that sources and magnetic fields are correlated with local large scale structure. And no single correlation can be significant when taking into account the ∼100,000 scan parameter combinations at all 6553 grid points.

A test for supergalactic structure, of energy-angle correlations, is not necessarily *apriori* obvious. The mean $\langle \tau_b \rangle$ inside equal solid angle bins of angular distance (*SGB_i*) from the supergalactic plane were used in the seven year data analysis. The single parameter chosen to test the supergalactic structure hypothesis was the curvature parameter, "*a*," of a parabolic fit $(ax^2 + bx + c)$ to the $\langle \tau_h \rangle$.

The curvature, "a", is simply the lowest order Taylor expansion term that can describe the symmetry around the SGP shown in simulation (Figure [3\(b\)](#page-4-0)). Greater correlation curvature means that the minimum negative correlation, and maximum positive correlation, averages are larger in magnitude. It also means that the minimum is closer to the supergalactic plane as shown in [\[10](#page-8-0)].

The large scale behavior of the correlation strength, τ_b , was used because it is not explicitly scanned for and contains more information by its sign (\pm) than the pre-trial significance. The pretrial significance of the correlations was not used so that the significance test was independent of the wedge scan for maximum energy-angle correlation significance.

The data significance of a supergalactic structure of energy-angle correlations the analysis was applied to the data, and the isotropic MC sets. The number of MC sets with a correlation curvature, *a*, greater than the data gives the probability that there is not a supergalactic structure of energy-angle correlations.

6. Results

The resulting data energy-angle correlations for seven years of data are shown in Figure $4(a)$. Individual correlations with the highest pre-trial significance are negative which means that there is a trend for the angular distance to increase with decreasing energy. This is the expectation for a grid point that happens to be near a UHECR source of magnetically scattered events. It can be seen that the negative τ_b correlations appear well correlated with the supergalactic plane.

Figure [4\(b\)](#page-6-0) shows the seven year data result of the mean τ_b correlation inside equal solid angle bins parallel to the supergalactic plane. The curvature was *a*=2.4×10−⁴ a minimum at −1.1 SGB. The data correlations have a very similar form to that of the supergalactic magnetic sheet simulation, shown in Figure [3\(b\),](#page-4-0) that has a slightly higher *a*=2.5×10−⁴ at −1.7 SGB. By applying this analysis to isotropic MC sets and counting the number of MC with an *a* parameter greater than data the post-trial significance of the supergalactic structure of multiplets was found to be \sim 4 σ [[10\]](#page-8-0).

Figure 4: Seven year data result. (a) Projection of the correlation strength τ_b for all grid points. Negative correlations expected for magnetic deflections are apparent around the supergalactic plane. Solid curves indicate the galactic plane (GP) in blue and supergalactic plane (SGP) in red. White and grey hexagrams indicate the Galactic center (GC) and anti-galactic center (Anti-GC) respectively. (b) Mean τ_b inside equal solid angle bins. The correlation curvature is $a=2.4\times10^{-4}$.

Figure 4(b) shows the 10 year mean τ_b correlation with no new scan for maximum correlation significances. The parabola curvature is $a=1.6\times10^{-4}$ and the minimum is at 1.2 SGB. It can be seen that the correlations are similar to the seven year result but the supergalactic structure does not appear quite as significant.

Figure 5: 10 year data result. (a) Projection of the correlation strength τ_b for all grid points. Negative correlations expected for magnetic deflections are apparent around the supergalactic plane. Solid curves indicate the galactic plane (GP) in blue and supergalactic plane (SGP) in red. White and grey hexagrams indicate the Galactic center (GC) and anti-galactic center (Anti-GC) respectively. (b) Mean τ_b inside equal solid angle bins. The correlation curvature is $a=1.6\times10^{-4}$.

6.1 Alternate Tests

There are other apparent correlation symmetries around the supergalactic plane that could be used to test a new independent data set, say TAx4 data for example [\[18\]](#page-9-0). Inside equal solid angle bins of supergalactic latitude the number of negative correlations, the mean correlation $sign(\tau_b) \times$ σ, and the mean estimated uniform magnetic field are examples.

The mean correlation $sign(\tau_b) \times \sigma$ is shown in Figure 6.1 and combines both values output by the τ_b correlation, the correlation sign and the scanned for (in the seven years of data) significance of the correlation. The curvature parameter for the seven year data set is $a = 0.0013$ and the ten year data set is $a = 0.0010$ which is less of a change between the two data sets than the mean τ_b .

Figure 6: The mean $sign(\tau_b) \times \sigma$ inside equal solid angle bins of SGL. (a) Seven years of data with a curvature parameter $a = 0.0013$. (b) Ten years of data with a curvature parameter $a = 0.0010$.

7. Summary

Intermediate-scale energy-angle correlations inside spherical cap sections, "wedges," were previously shown, using seven years of Telescope Array data, to have a \sim 4 σ correlation with the supergalactic plane. Even though the ten years of data update does not appear to be as significant this may be possible evidence of large scale magnetic diffusion of ultra-high energy cosmic rays from their sources correlated with the local large scale structure. The highest significance single energy-angle correlation has increased from a pre-trial 4.6σ significance to pre-trial 5.1σ with no new scan of wedge parameters. This correlations origin point is 10◦ from M82 and lies over the TA Hotspot. Confirmation of this result may be done once sufficient data is collected by the Telescope Array expansion to TAx4 [\[18](#page-9-0)].

Acknowledgments

The Telescope Array experiment is supported by the Japan Society for the Promotion of Science(JSPS) through Grants-in-Aid for Priority Area 431, Specially Promoted Research JP21000002, Scientific Research (S) JP19104006, Specially Promoted Research JP15H05693, Scientific Research (S) JP15H05741, Science Research (A) JP18H03705, and for Young Scientists (A) JPH26707011; by the joint research program of the Institute for Cosmic Ray Research (ICRR), The University of Tokyo; by the U.S. National Science Foundation awards PHY-0601915, PHY-1404495, PHY-1404502, and PHY-1607727; by the National Research Foundation of Korea (2016R1A2B4014967, 2016R1A5A1013277, 2017K1A4A3015188, 2017R1A2A1A05071429) ; by the Russian Academy of Sciences, RFBR grant 16-02-00962a (INR), IISN project No. 4.4502.13, and Belgian Science Policy under IUAP VII/37 (ULB). The foundations of Dr. Ezekiel R. and Edna Wattis Dumke, Willard L. Eccles, and George S. and Dolores Doré Eccles all helped with generous donations. The State of Utah supported the project through its Economic Development Board, and the University of Utah through the Office of the Vice President for Research. The experimental site became available through the cooperation of the Utah School and Institutional Trust Lands Administration (SITLA), U.S. Bureau of Land Management (BLM), and the U.S. Air Force. We appreciate the assistance of the State of Utah and Fillmore offices of the BLM in crafting the Plan

of Development for the site. Patrick Shea assisted the collaboration with valuable advice on a variety of topics. The people and the officials of Millard County, Utah have been a source of steadfast and warm support for our work which we greatly appreciate. We are indebted to the Millard County Road Department for their efforts to maintain and clear the roads which get us to our sites. We gratefully acknowledge the contribution from the technical staffs of our home institutions. An allocation of computer time from the Center for High Performance Computing at the University of Utah is gratefully acknowledged.

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